### Data Set

<table>
<thead>
<tr>
<th>Minimum Number</th>
<th>Maximum Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

### Range

\[
\text{Range} = \text{Max} - \text{Min} = 4 - 1 = 3
\]

*Range is a value, not an interval*

**Range:** This is the difference between the largest (Max) and smallest (Min) values.

- One measure of dispersion-

---

**Dispersion Indices:**

- **How Far the point deviates from the center of the data.**
- **Dispersion Indices look at all the points.**

**Sand dropped on the table**

---

**For Groups**

\[
\begin{align*}
G_1 & & G_2 & & G_3 \\
- & & - & & - \\
- & & - & & - \\
- & & - & & - \\
- & & - & & - \\
\end{align*}
\]

The **Average Range**

\[
\frac{R_1 + R_2 + R_3}{3} = \bar{R}
\]
Dispersion Indices

Exploring an Example Set of Raw Deviations

I could find the average deviation and call it the Standard Deviation

\[ \delta_1 = x_{10} - \text{Center} \]

\[ \delta_1, \delta_2, \delta_3, \ldots, \delta_N \]
Dispersion Indices

With a Normal Curve, the Mean, Median and Mode are all at the same point.

If you don’t know what $X_{18}$ is, your best estimate is the mean. The model measurement is the mean. (it minimizes your uncertainty)

$$\delta = X_{18} - \mu$$

We are contrasting each measurement to the Model Condition
\[ \mu = 10 \]
\[ X_i = 12 \]
\[ 12 - 10 = +2 \]
\[ X_i - \mu = \delta \]

\[ (X_i - \mu)^2 = \delta^2 \]

\[ \sum (x - \mu)^2 \]

The Quadratic Effect
*The greater the deviation the more weight it carries*
## Dispersion Indices

### Understanding and Interrogating the Sum-of-Squares (SS)

#### Table: Sum-of-Squares Calculation

<table>
<thead>
<tr>
<th>( X )</th>
<th>( x - \overline{x} )</th>
<th>( (x - \overline{x})^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 - 3 = -2</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>2 - 3 = -1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>3 - 3 = 0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>4 - 3 = +1</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>5 - 3 = +2</td>
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</tr>
</tbody>
</table>

\[
\frac{\sum (X)}{n} = \frac{15}{5} = 3 = \overline{x}
\]

Balance

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Understanding and Interrogating the Sum-of-Squares (SS)
Dispersion Indices

Using the Variance to Assess Dispersion

<table>
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\[ \frac{\sum(X)}{n} = \frac{15}{5} = 3 = \bar{x} \quad 0 \quad 10 = \sum(x - x)^2 = \text{Sum of Squares or "SS"} \]

\[ \text{Variance} = S^2 = \frac{\sum(x - \bar{x})^2}{n - 1} = \frac{\text{SS}}{n - 1} \]

\[ S^2 = \frac{\sum(x - \bar{x})^2}{n - 1} = \frac{\text{SS}}{n - 1} = \frac{10}{4} \]

\[ S = \sqrt{S^2} \]
Dispersion Indices

Variance = $S^2 = \frac{SS}{n - 1} = \frac{10}{4} = 2.50$

The Standard Deviation is: $S = \sqrt{S^2}$

Therefore the Standard Deviation or “S” is: $S = \sqrt{2.50} = 1.58$

Notation for the Standard Deviation:
- Upper Case “S” relates to the Population
- Lower Case “s” relates to a Sample
- $\sigma$ relates to the Population
- $\hat{\sigma}$ relates to a Sample. A Estimated or Postulated value

$$\sigma = \sqrt{\frac{\sum(x - \mu)^2}{N}} \quad \text{Population}$$

$$\hat{\sigma} = \sqrt{\frac{\sum(x - \bar{x})^2}{n - 1}} \quad \text{Sample}$$

Used in our Example
\[
\hat{\sigma} = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}
\]

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\[
\frac{\sum(x)}{n} = \frac{15}{5} = 3 = \bar{x}
\]

\[
\pm 3\hat{\sigma} \rightarrow 99.73\% \text{ area under the curve}
\]
Dispersion Indices

\[ (x - \bar{x})^2 \]

5 Times

\[ \sum (x - \bar{x})^2 \]

Sum of Squares

\[ \hat{\sigma} = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}} = 1.58 \]

Variance

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Geometric Visualization of a Variance